Sample size and power calculations for open cohort stepped wedge designs

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Longitudinal Cluster Randomised Trials

For example:

	Period 1	Period 2	Period 3	Period 4
Sequence 1	Control	Intervention	Intervention	Intervention
Sequence 2	Control	Control	Intervention	Intervention
Sequence 3	Control	Control	Control	Intervention

- Clusters measured repeatedly over time.
- But what about participants within clusters?

Longitudinal CRT Sampling Structures (due to Copas et al.)

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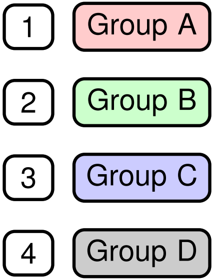
Continuous recruitment: each participant appears in exactly one period.



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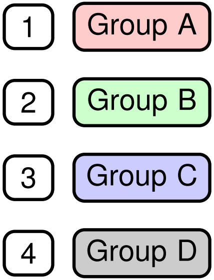
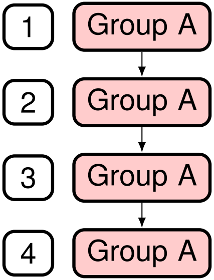


Closed cohort: each participant appears in each period.



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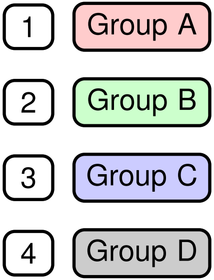

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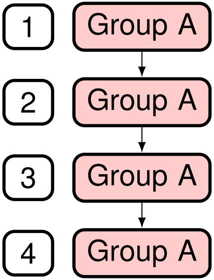

Open cohort: participants appear in variable numbers of periods.

Many different ways in which this can happen.

Longitudinal CRTs and open cohorts

Note: m **subjects provide continuous outcomes in each cluster in each period.**

$$Y_{kti} = \beta_t + \theta X_{kt} + C_k + CP_{kt} + \eta_{ki} + \epsilon_{kti}$$

$$\eta_{ki} \sim N(0, \sigma^2_\eta), \quad \epsilon_{kti} \sim N(0, \sigma^2_\epsilon), \quad C_k \sim N(0, \sigma^2_C), \quad CP_{kt} \sim N(0, \sigma^2_{CP})$$

- subject $i = 1, \ldots, m$; period $t = 1, \ldots, T$; cluster $k = 1, \ldots K$.

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Collapse to cluster-period means:

$$\frac{1}{m}\sum_{i=1}^{m} Y_{kti} = \bar{Y}_{kt\bullet} = \beta_t + \theta X_{kt} + C_k + CP_{kt} + \eta_{k\bullet} + \epsilon_{kt\bullet}$$

$$var\left(\bar{Y}_{kt\bullet}\right) = \sigma^2_C + \sigma^2_{CP} + \frac{\sigma^2_\eta}{m} + \frac{\sigma^2_\epsilon}{m}, \quad cov\left(\bar{Y}_{kt\bullet}, \bar{Y}_{ks\bullet}\right) = \sigma^2_C + \sigma^2_\eta \frac{n_k(t,s)}{m^2}$$

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$n_k(t,s)$: the number of subjects in cluster k who appear in both period t and period s.

Participant attrition/churn

Churn rate from period t to period s in cluster k:

- the proportion of participants in period t who **do not** appear in period s

$$\chi_k(t, s) = 1 - \frac{n_k(t, s)}{m}$$

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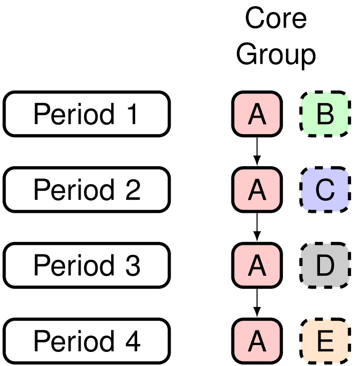
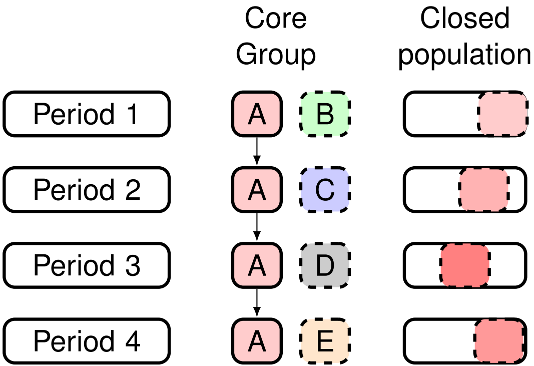
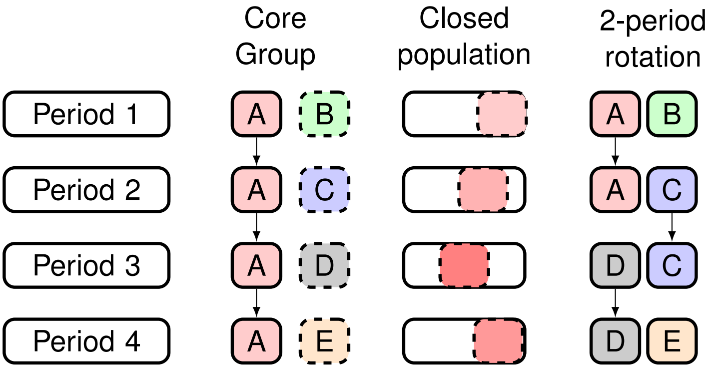
- Open cohort: $\chi_k(t, s)$ depends on the **precise open cohort sampling scheme.**

Open cohort sampling schemes



Open cohort sampling schemes




Core Group
Closed population
2-period rotation
Period 1
Period 2
Period 3
Period 4
A
B
A
C
A
D
A
E
A
B
A
C
D
C
D
E

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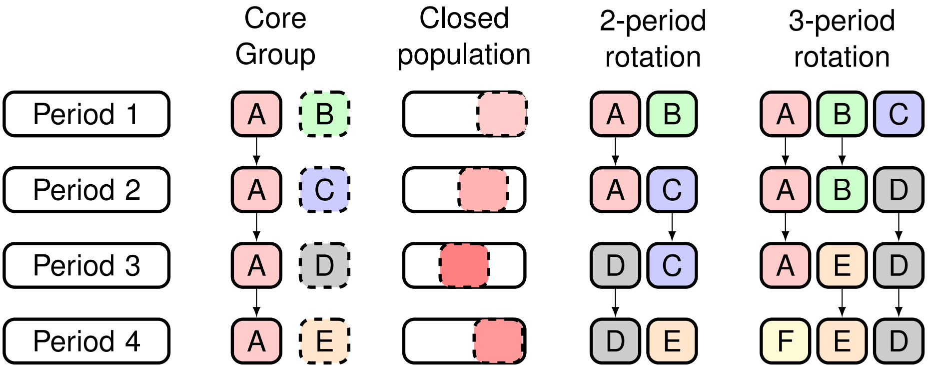

Churn rates for open cohort sampling schemes

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p-period rotation: $\chi_k(t,s) = \frac{|t-s|}{p}$ for $|t-s| \leq p$ and 1 for $|t-s| > p$

But how can we calculate sample size?

$$\sigma^2 = \sigma^2_C + \sigma^2_{CP} + \sigma^2_\eta + \sigma^2_\epsilon, \quad \rho = \frac{\sigma^2_C + \sigma^2_{CP}}{\sigma^2}, \quad \pi = \frac{\sigma^2_C}{\sigma^2_C + \sigma^2_{CP}}, \quad \tau = \frac{\sigma^2_\eta}{\sigma^2_\eta + \sigma^2_\epsilon}$$

- σ^2: the total variance;
- ρ: the usual intra-cluster correlation;
- π: the cluster autocorrelation;
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- Hooper et al. (2016) provided design effects for repeated cross-sections and closed cohort designs.
- Feldman and McKinlay (1994) discussed designs with “random overlap”:
  - Their recommendation: vary the participant autocorrelation to account for the overlap of samples.

A design effect for open cohorts ($\chi_k(t,s) = \chi$)

Required number of clusters:

$$K_L = DE(r) \times [1 + (m-1)\rho] \frac{n_i}{m}$$

- n_i: total number of participants required for an individually-randomised trial
- m: number of participants measured in each cluster in each period
- $DE(r)$ depends on the design schematic:

$$DE(r) = \frac{1}{4} \frac{K^2(1-r)[1+(T-1)r]}{KX_{\bullet\bullet} - \sum_{t=1}^{T}(X_{\bullet t})^2 + \left[(X_{\bullet\bullet})^2 + K(T-1)X_{\bullet\bullet} - (T-1)\sum_{t=1}^{T}(X_{\bullet t})^2 - K\sum_{k=1}^{K}(X_{k\bullet})^2\right]}$$

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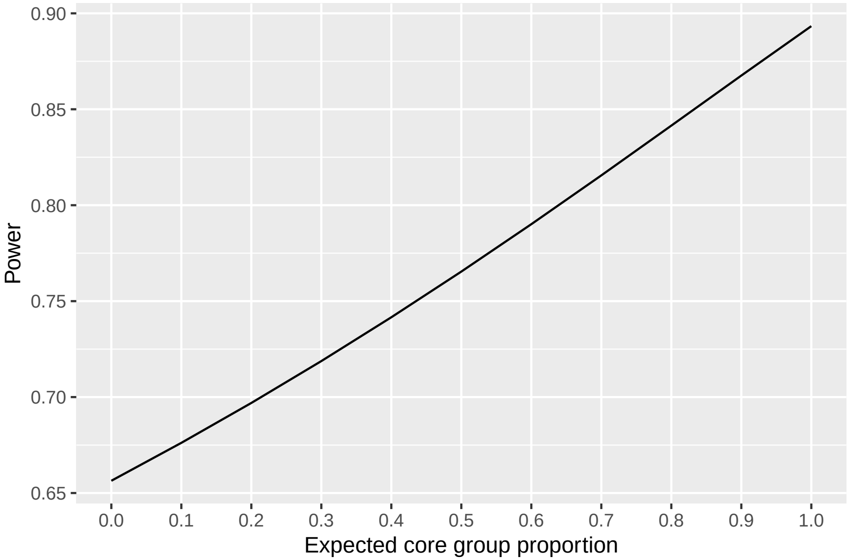
Girls on the Go! example

	Period 1	Period 2	Period 3	Period 4
School 1	n=10	n=10	n=10	n=10
School 2	n=10	n=10	n=10	n=10
School 3	n=10	n=10	n=10	n=10
School 4	n=10	n=10	n=10	n=10
School 5	n=10	n=10	n=10	n=10
School 6	n=10	n=10	n=10	n=10

- A stepped wedge trial in Aussie primary schools to evaluate a programme to increase the self-esteem of young women, measured on continuous scale.
- $\rho = 0.33$, $\pi = 0.9$, $\tau = 0.7$, total variance 25, difference in means of interest 2 units.

What if this design had a core group open cohort sampling scheme?

Power with varying core group proportions



Take home messages

- The open cohort design effect unifies the single measurement and closed cohort results.
- Conservative assumption: one measurement per subject.
    - But researchers should account for the actual sampling scheme.
- We considered three types of open cohort sampling schemes - but more are available!

Shiny app available:

`https://monash-biostat.shinyapps.io/OpenCohort/`